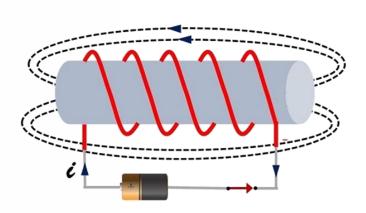


What is Self Induction?





Unit Two – Electricity

Chapter 9 – Self Induction

ACADEMY

Prepared & Presented by: Mr. Mohamad Seif



OBJECTIVES

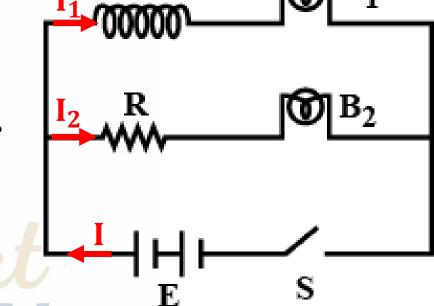
1 Evidence of Self induction

2 Inductance of the coil

3 Derive the expression of the electromotive force e.m.f

The two parallel branches shown contain two identical lamps B_1 and B_2 .

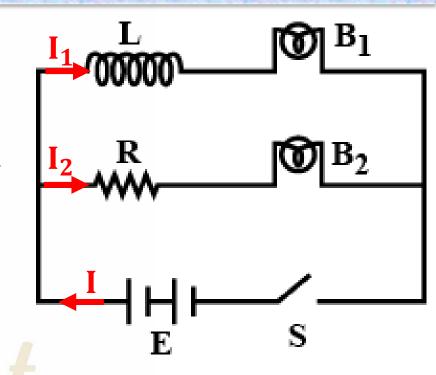
The resistor has the same resistance of the coil.



The two currents in the two branches are expected to be equal and the two lamps are expected to glow in the same manner.

First case: Close the switch K:

We notice that Lamp B_2 glows instantly, while B_1 glows gradually with a noticeable delay in B_1 with respect to B_2 . Then B_1 glows as bright as B_2 .



Second case: Open the switch K:

We notice that Lamp B_2 turns off instantly, while the brightness of B_1 decrease gradually with a noticeable delay of time. Then B_1 turns off.

First case: Close the switch K

In a resistor the current increases suddenly from zero to a

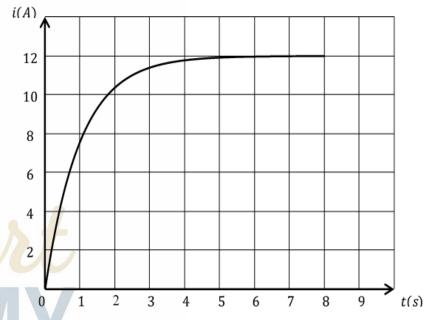
value I.

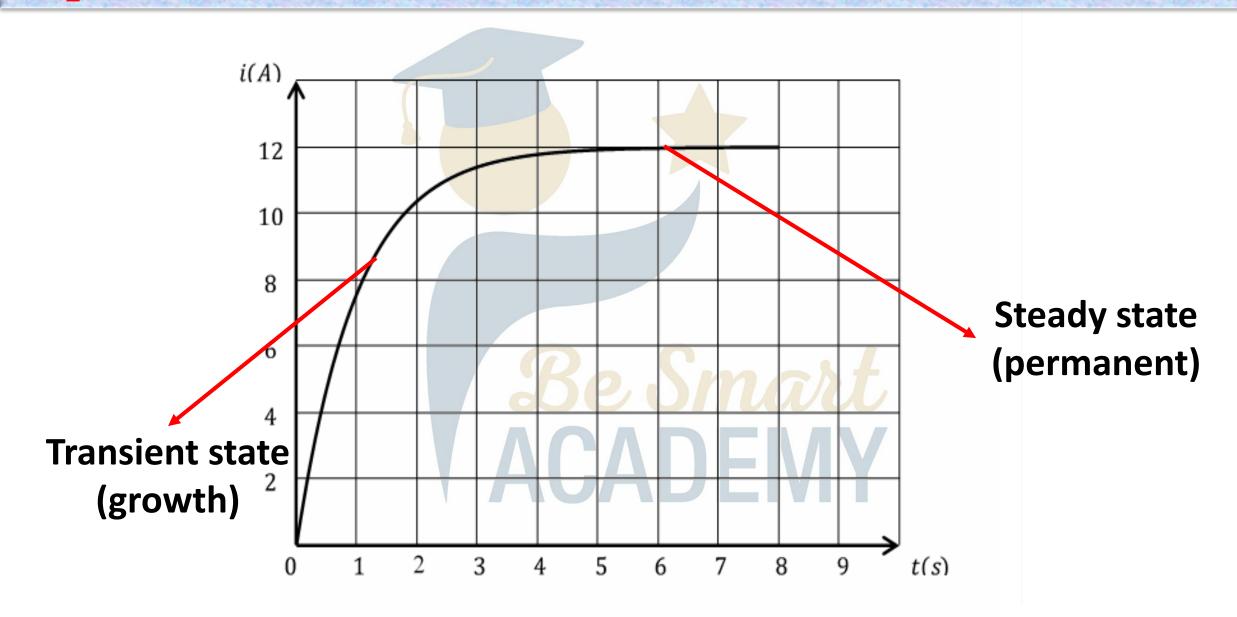
In the coil the current grows progressively from zero to reach the same value I in the steady state.

In fact, a variable current traverses

the coil, then a variable magnetic field

is created inside the coil.





- Due to this current, the coil is crossed by a variable flux. The variable magnetic flux leads to an induced electromotive force, then an induced current traverses the coil.
- According to Lenz's law, the induced current opposes the increase of the main current. As a result, the main current takes a period of time to reach its maximum value I.

This phenomenon is called self induction.

The Flux is called self or proper flux

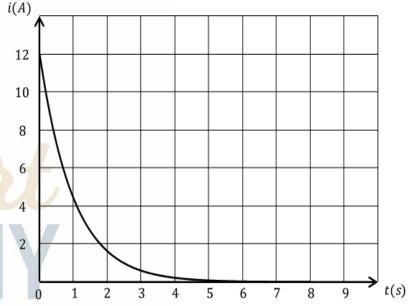
Electromotive force is called self indued electromotive force

Second case: open the switch K

In the resistor the current decreases suddenly from I to zero, while in coil the current decreases progressively from I to zero.

In fact, a variable current traverses the coil, and then a variable magnetic field is created inside the coil.

Due to this current, the coil is crossed by a variable magnetic flux.



The variable magnetic flux leads to an induced electromotive force, then an induced current traverses the coil.

According to Lenz's law the induced current opposes the decrease of the main current, for this reason the main current takes an elapse of time to reach zero.

This phenomenon is called self induction.

1 ACADEMY

Inductance of the coil

The self –flux (\emptyset) crossing a coil depends on the current flowing in it.

The The self –flux is given by the following expression:

$$\emptyset = L.i$$

L: characteristic property of the coil, called inductance, in Henry (H).

Ø: self flux in Wb.

i: the electric current traversing the coil in ampere (A).

Inductance of the coil

The value of the inductance of long solenoid:

In this chapter, it is preferable to make the positive sense in the direction of the current, thus the angle $\theta = 0$

$$\emptyset = NBScos(0)$$

$$\emptyset = NBS$$

Inductance of the coil

$$\emptyset = Li$$

$$L = \frac{\emptyset}{i}$$

$$L=\frac{NBS}{i}$$

$$L = \frac{NS\left[4\pi \times 10^{-7} \frac{Ni}{l}\right]}{i}$$

$$L = \frac{4\pi \times 10^{-7} N^2 S}{l}$$

(must be derived before use)

Smart DEMY

Expression of the self-induced electromotive force e.m.f

The electromotive force "e" is the negative of the derivative of the self flux:

$$e = -\frac{d\psi}{dt}$$

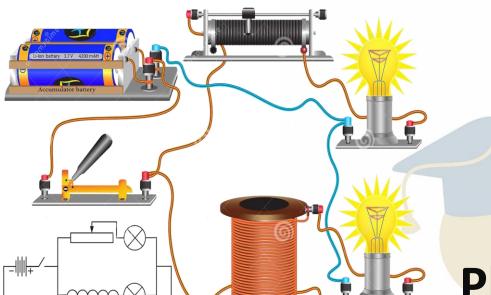
$$e = -\frac{d(Li)}{dt}$$

$$e = -\frac{Ldi}{dt}$$

$$e = -\frac{Ldi}{dt}$$

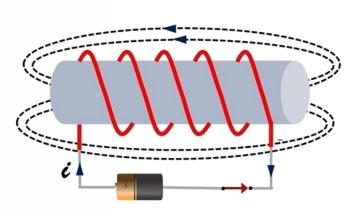
Since L >0 then e and $\frac{di}{dt}$ are of opposite signs





What is **Self Induction?**

Physics – Grade 12



Unit Two – Electricity

Chapter 9 – Self Induction

ACADEMY

Prepared & Presented by: Mr. Mohamad Seif



OBJECTIVES

1 Derive the voltage of the coil

2 Specify the Role of the coil

3 Derive the distribution of the power in the circuit

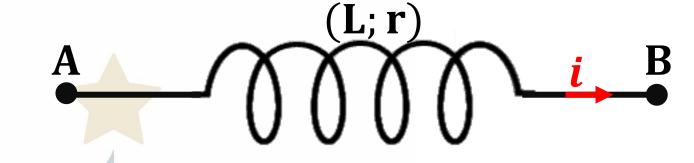
Voltage of the coil

Ohm's law case of a coil:

$$u_{AB} = ri - e$$

But
$$e = -L \frac{di}{dt}$$

$$u_{AB} = ri - \left[-L \frac{di}{dt} \right]$$



Voltage of the coil

$$u_{AB} = ri + L\frac{di}{dt}$$

Special cases:

Case 1: Coil of negligible resistance (r = 0):

$$u_{AB} = L \frac{di}{dt}$$

Case 2: at steady state: The current is constant

$$\frac{di}{dt} = 0$$

$$ACAD_{u_{AB}} Y_i$$

A coil carrying a current *i* is oriented positively from A to B:

Case 1: the coil acts as a receiver

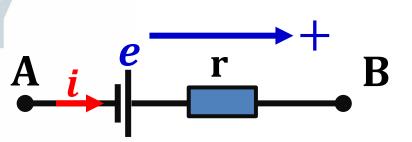
The electric current increases with time:

$$e = -L\frac{di}{dt} < 0 \qquad e. i < 0$$



The electric current decreases with time:

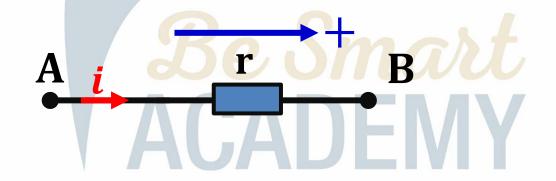
$$e = -L\frac{di}{dt} > 0 \qquad \qquad e. i > 0$$



Case 3: the coil acts as a resitor

The electric current is constant:

$$e = -L\frac{di}{dt} = 0$$
 \Rightarrow $u_{AB} = ri$ \Rightarrow Steady state

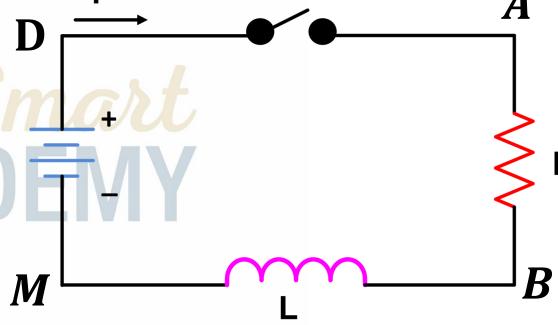


Application 1: A coil of inductance L = 10mH and of internal resistance $r = 5\Omega$ is connected in series with a resistor of resistance $R = 15\Omega$, a switch K, and an ideal battery of electromotive force E = 12V as shown in the adjacent figure.

1) At the instant $t_0 = 0$:

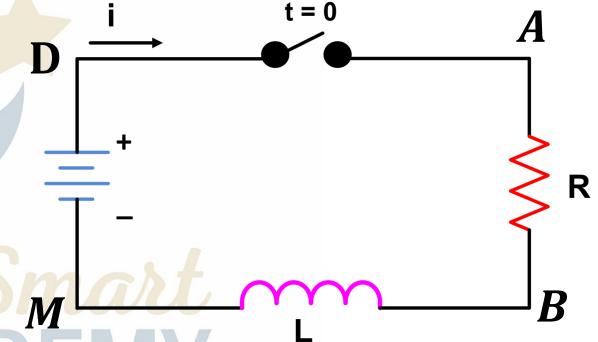
a. Indicate the value of the current *i* in the circuit.

At $t_0 = 0$, $i_0 = 0$



b. Apply the law of addition of voltage to determine the voltage u_{BM} across the coil.

$$u_{DM} = u_{DA} + u_{AB} + u_{BM}$$
 $E = 0 + Ri + u_{BM}$
 $E = 0 + R(0) + u_{BM}$
 $u_{BM} = E = 12V$



c. Calculate the value of the self-induced e.m.f "e".

$$u_{BM} = ri - e \implies 12V = r(0) - e \implies e = -12V$$

d. Deduce the value of $\frac{di}{dt}$

$$e = -L\frac{di}{dt}$$

$$-12 = -(10 \times 10^{-3}) \frac{di}{dt}$$

$$\frac{di}{dt} = -12$$

$$\frac{di}{dt} = -10 \times 10^{-3}$$

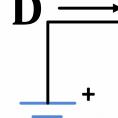
$$\frac{di}{dt} = 1200A/s$$

2) At instant t_1 , $\frac{di}{dt} = 441.455 A/s$, determine the current i.

$$u_{DM} = u_{DA} + u_{AB} + u_{BM} \quad \Longrightarrow$$

$$E = 0 + Ri + ri + L\frac{di}{dt}$$

$$E = (R+r)i + L\frac{di}{dt}$$



$$12 = (15+5)i + 10 \times 10^{-3} \times 441.455 =$$

$$12 = 20i + 4.41455$$

$$12 - 4.41455 = 20i$$

$$7.58545 = 20i$$

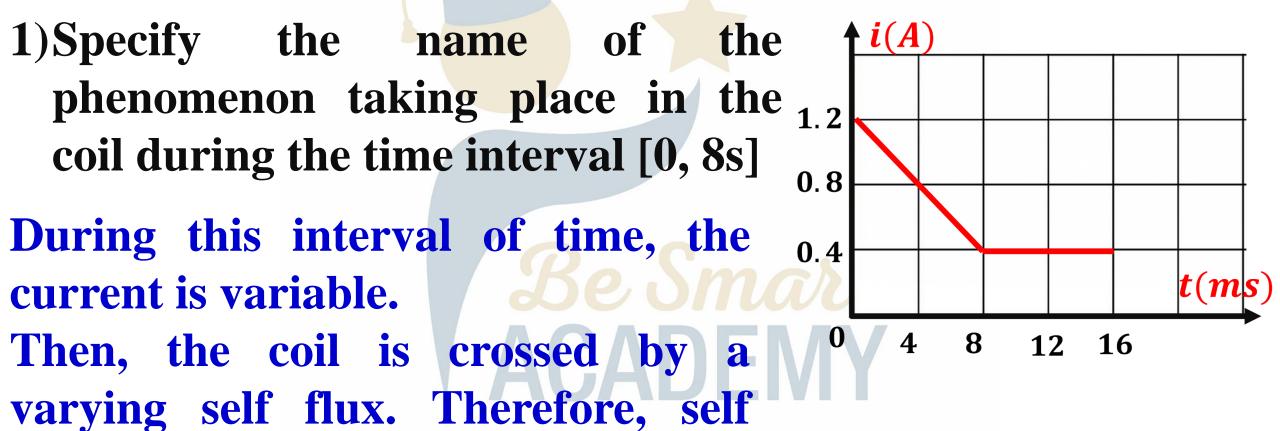


$$i = 0.379A$$

Application 2:

induction is taking place

A coil of inductance L = 20mH carries an electric current whose value varies with time according to the graph below:



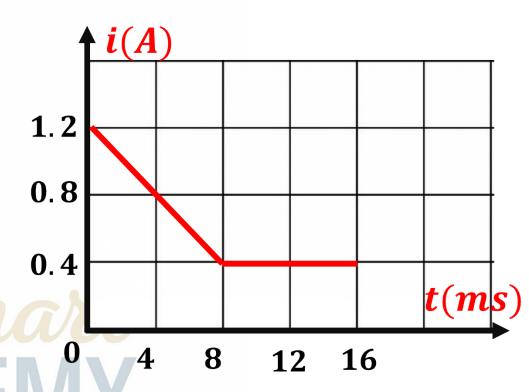
2) Specify the role of the coil in each interval of time.

For the time interval [0, 8s]

The current decreases with time:

$$\frac{di}{dt} < 0 \qquad \Rightarrow \qquad e = -L \frac{di}{dt} > 0$$

$$e. i > 0$$



The coil acts as a generator

2) Specify the role of the coil in each interval of time.

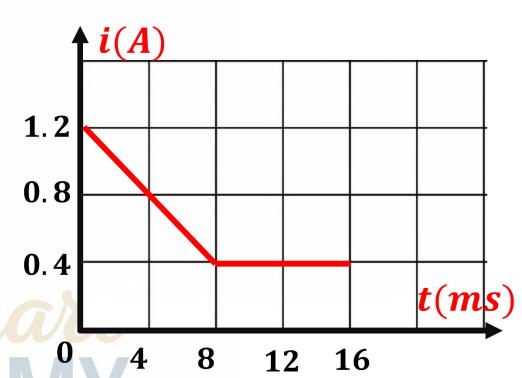
For the time interval [8s, 16s]

The current is constant:

$$\frac{di}{dt} = 0$$



$$\frac{di}{dt} = 0 \quad \Longrightarrow \quad e = -L\frac{di}{dt} = 0$$



The coil acts as a resistor

3) Determine the value of the electromotive force "e" during each interval of time.

For the time interval [0, 8s]

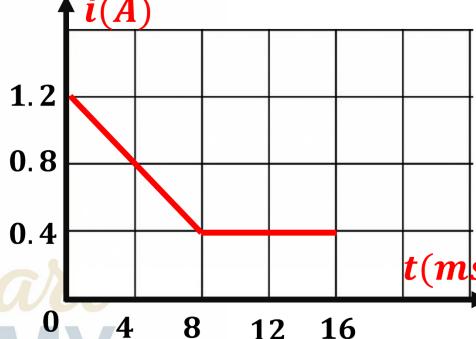
 $\frac{di}{dt}$ is the slope of the St. line

$$\frac{di}{dt} = \frac{0.4 - 1.2}{(8 - 0) \times 10^{-3}} = -100A/s$$

$$e = -L\frac{di}{dt} = -20 \times 10^{-3} \times (-100)$$



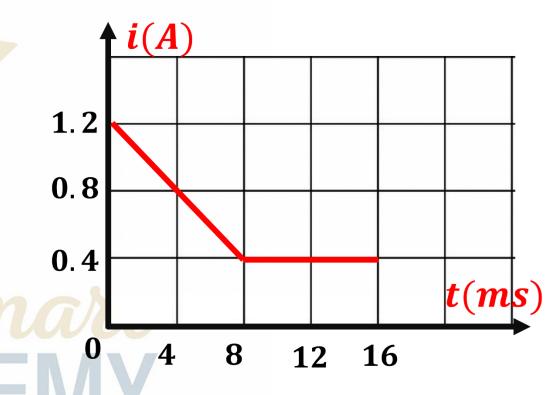
$$e = 2V$$



For the time interval [8s, 16s]

The current is constant then
$$\frac{di}{dt} = 0$$

$$e = -L\frac{di}{dt} = -20 \times 10^{-3} \times (0)$$



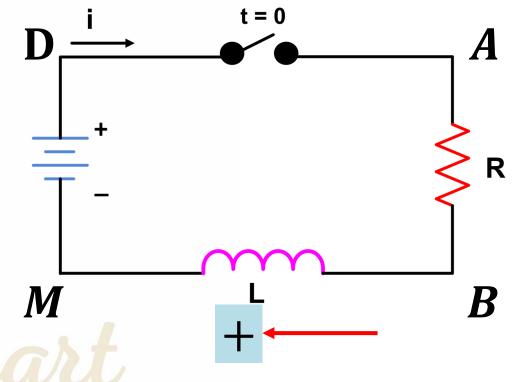
Power distribution in the circuit

A coil is oriented positively as shown:

$$u_{BM} = ri + L\frac{di}{dt}$$

Multiply the equation by (i)

$$i. u_{BM} = ri^2 + Li \frac{di}{dt}$$



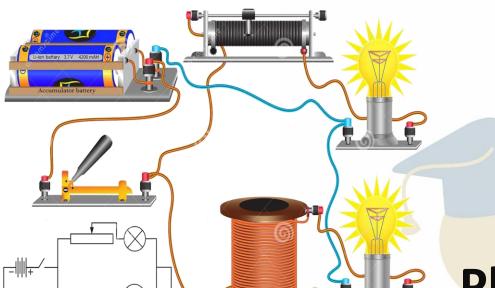
$$P_{total} = P_{lost} + P_{magnetic}$$

Power distribution in the circuit

$$i. u_{BM} = ri^2 + Li \frac{di}{dt}$$

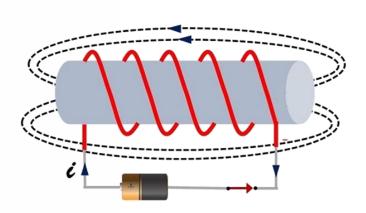
- $P_{total} = i.u_{BM}$: is the total electric power received by the coil during the growth process.
- $P_{lost} = ri^2$: lost power due to Joule's effect in the coil
- $P_{magnetic} = Li \frac{di}{dt}$: magnetic power (rate of storing magnetic energy)





What is Self Induction?





Unit Two – Electricity

Chapter 9 – Self Induction

ACADEMY

Prepared & Presented by: Mr. Mohamad Seif



OBJECTIVES

Derive the expression of the magnetic energy

ACADEMY

Magnetic energy stored in the coil.

The expression of the instantaneous power of an energy conversion is:

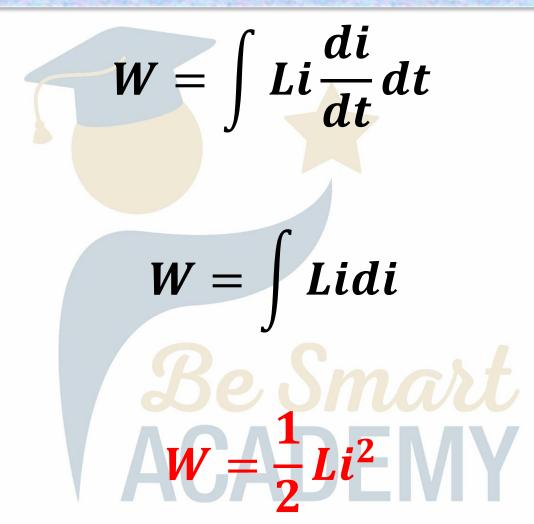
$$P = \frac{dW}{dt}$$

$$dW = Pdt$$

$$\int dW = \int Pdt$$

$$W = \int Li \frac{di}{dt} dt$$

Magnetic energy stored in the coil.



Application 3: The circuit below includes a coil of inductance L = 1mH and internal resistance $r = 2\Omega$, a resistor of resistance $R = 8\Omega$, an ideal battery of electromotive force E = 10V, and a switch K.

1. Show that at $t_0 = 0$ the current sent by the battery is zero.

$$W = \frac{1}{2}Li^{2}$$
At $t_{0} = 0$, the magnetic energy stored in the coil is zero then:
$$0 = \frac{1}{2}Li_{0}^{2}$$

$$i_{0} = 0$$

2. Calculate the value of the maximum current I_m at steady state.

At steady state:

$$m = \frac{E}{r + R}$$

$$I_m = \frac{10}{2+8}$$

$$I_m = 1A$$

3. Deduce the maximum magnetic energy stored in the coil.

$$W_{max} = \frac{1}{2}Li^2$$

$$W_{max} = \frac{1}{2}Li^2$$
 $W_{max} = \frac{1}{2}(1 \times 10^{-3})(1)^2$

$$W_{max} = 5 \times 10^{-4} J$$

4. The electric energy delivered by the battery between t_0 and t_1 is $8W_{max}$. Determine the dissipated energy in the circuit due to Joule's effect.

$$W_g = W_{max} + W_{heat}$$
 8 $W_{max} = W_{max} + W_{heat}$

$$W_{heat} = 7W_{max}$$

$$W_{heat} = 7 \times 5 \times 10^{-4}$$

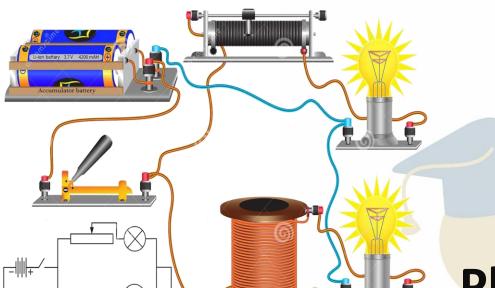
$$W_{heat} = 35 \times 10^{-4} J$$

- 5. Specify the role of the coil between $t_0 = 0$ and t_1 .
 - The current flowing in the coil increases from zero to its maximum value at the steady state:

ACADEMY

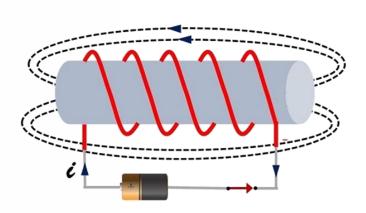
- Then the coil is storing magnetic energy:
- Then the coil acts as a receiver.





What is Self Induction?





Unit Two – Electricity

Chapter 9 – Self Induction

ACADEMY

Prepared & Presented by: Mr. Mohamad Seif



OBJECTIVES

study the growth of the current in RL circuit

ACADEMY

Consider the following circuit:

The Voltage u_{AC} of the generator is square voltage:

$$u_{AC} = \begin{cases} u_{AC} = E \dots 0 < t < \frac{T}{2} \\ u_{AC} = 0 \dots \frac{T}{2} < t < T \end{cases}$$

$$ACADE^{C}$$

$$ACADE^{C}$$

$$ACADE^{C}$$

$$ACADE^{C}$$

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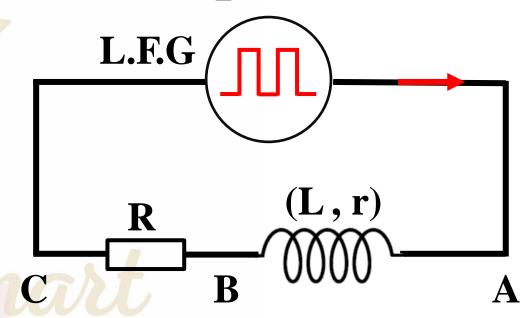
Growth of the current: $u_{AC} = E \dots 0 < t < \frac{1}{2}$

Apply law of addition of voltages:

$$u_{AC} = u_{AB} + u_{BC}$$

$$E = ri + L\frac{di}{dt} + Ri$$

$$E = (R+r)i + L\frac{di}{dt}$$



Differential equation in terms of current i

The solution of the differential equation is: $i = I(1 - e^{-\frac{t}{\tau}})$

$$i = I(1 - e^{-\frac{\iota}{\tau}})$$

Where
$$\tau = \frac{L}{(r+R)}$$
 And $I = \frac{E}{(r+R)}$

And
$$I = \frac{E}{(r+R)}$$

At t = 0:

$$i = I \left(1 - e^{-\frac{0}{\tau}} \right)$$

$$i = J \left(1 - e^{-\frac{0}{\tau}} \right)$$

$$i = 0$$

At
$$t = \tau$$

$$i = I(1 - e^{-\frac{\tau}{\tau}})$$

$$i = I(1 - e^{-1})$$



$$i = I(1 - 0.37)$$

$$i = 0.63I = 63\%I$$

 $t=\tau$: Time constant is the interval of time after which the current reaches 63% of its max value during growth process.

At
$$t = 5\tau$$

$$i = I \left[1 - e^{-\frac{5\tau}{\tau}} \right]$$

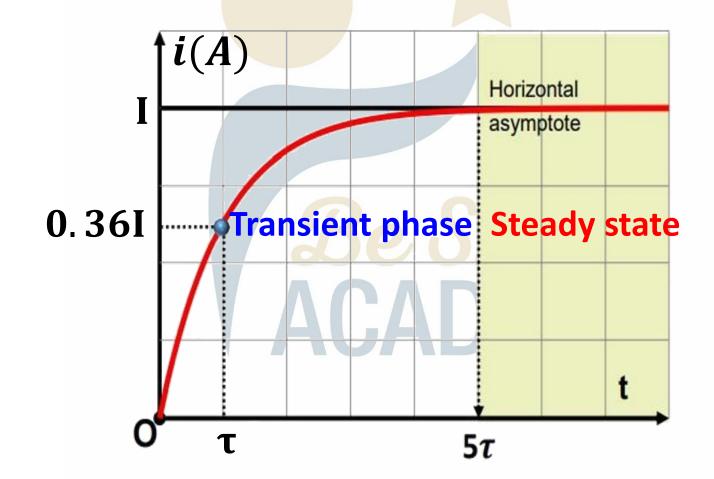
$$u_{\mathcal{C}} = I(1 - e^{-5})$$

$$u_C = I(1 - e^{-5})$$
 $i = I(1 - 0.0067)$

$$i = 0.99I \approx I$$

At $t = 5\tau$, the steady state is attained and the coil acts as a resistor of resistance r.

t = 0	$t = \tau$	$t=5\tau$
i = 0	i=0.63I	i = I



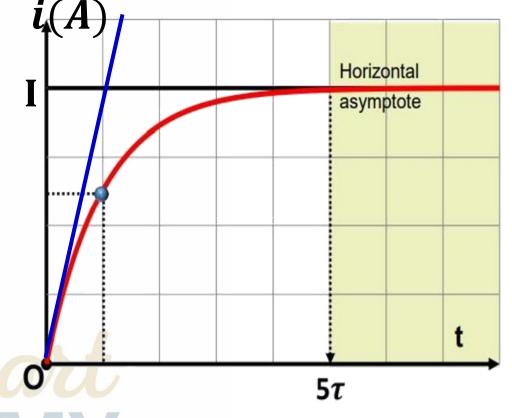
Time constant using Tangent method:

First method:

Tangent to i = f(t) at t = 0 is drawn

The slope of the tangent at t = 0 is $\frac{di}{dt}$

$$i = I(1 - e^{-\frac{t}{\tau}}) \implies \frac{di}{dt} = I\left[\frac{1}{\tau}\right]e^{-\frac{t}{\tau}}$$



At
$$t = 0$$
:

$$\frac{di}{dt} = I \left[\frac{1}{\tau} \right] e^{-\frac{0}{\tau}} \implies \text{slope} = \frac{di}{dt} = \frac{I}{\tau}$$



slope =
$$\frac{ai}{dt} = \frac{i}{\tau}$$

In
$$\triangle$$
 OHK: $tan(\alpha) = \frac{HK}{OH} = \frac{I}{OH}$

$$slope of tangent$$

$$\frac{I}{\tau} = \frac{I}{OH}$$

$$\tau = OH$$

$$I = \frac{I}{OH}$$

$$\frac{I}{OH} = \frac{I}{OH}$$

Therefore, the tangent to i at t=0 meets the asymptote in a point of abscissa au

Second method:

The equation of tangent to i at t = 0 I

is: i = at

Where a is the slope of the tangent at

$$t = 0$$

$$E = I(1 - e^{-\frac{t}{\tau}})$$



$$i = I(1 - e^{-\frac{t}{\tau}}) \implies \frac{di}{dt} = I\left[\frac{1}{\tau}\right]e^{-\frac{t}{\tau}}$$

At
$$t = 0$$
:

$$=I\left|rac{1}{\tau}
ight|e^{-rac{0}{ au}}$$



slope =
$$\frac{di}{dt} = \frac{I}{\tau}$$

Horizontal

asymptote

5τ

The equation of tangent is:
$$i = at$$

$$i = \frac{I}{\tau} \cdot t$$

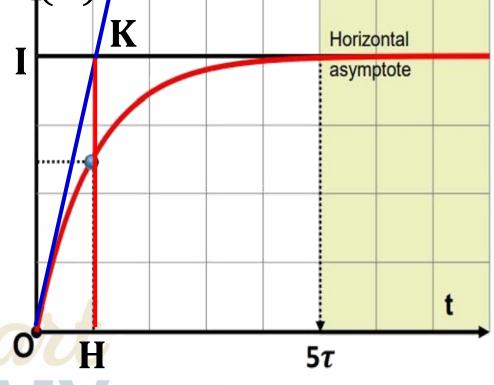
The equation of asymptote to i is: i = I.

The tangent and asymptote intersect

at point K. Then:

$$i_{tangent} = i_{asymptote}$$

$$\frac{I}{\tau}.t = I \Rightarrow ACA = T$$



Therefore, the tangent to i=f(t) at t=0 meets the asymptote in a point of abscissa τ .

Determination of I and τ :

$$E = R_{eq}i + L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{I}{\tau} \cdot e^{-\frac{t}{\tau}}$$

$$i = I(1 - e^{-\frac{t}{\tau}})$$

$$\frac{di}{dt} = \frac{I}{\tau} \cdot e^{-\frac{t}{\tau}}$$
 Substitute i and $\frac{di}{dt}$ in differential equation.

$$E = R_{eq}I(1 - e^{-\frac{t}{\tau}}) + L.\frac{I}{\tau}.e^{-\frac{t}{\tau}}$$

$$E = R_{eq}I - R_{eq}Ie^{-\frac{t}{\tau}} + \frac{LI}{\tau}.e^{-\frac{t}{\tau}}$$

$$E = R_{eq}I - R_{eq}Ie^{-\frac{t}{\tau}} + \frac{LI}{\tau} \cdot e^{-\frac{t}{\tau}}$$

$$E + R_{eq}Ie^{-\frac{t}{\tau}} = R_{eq}I + \frac{LI}{\tau} \cdot e^{-\frac{t}{\tau}}$$

By identification we get:

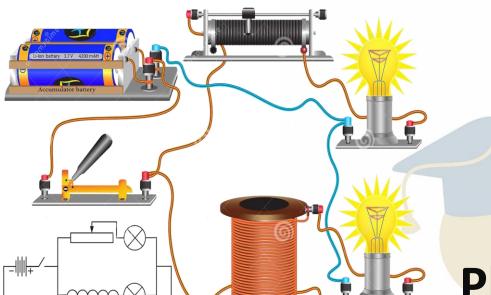
$$E = R_{eq}I$$

$$AGADEMY$$

$$R_{eq}I$$

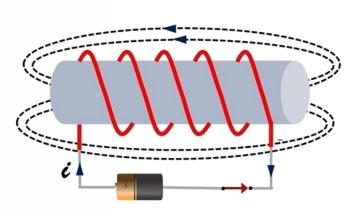
$$R_{eq}I = \frac{LI}{\tau}$$





What is **Self Induction?**

Physics – Grade 12



Unit Two – Electricity

Chapter 9 – Self Induction

ACADEMY

Prepared & Presented by: Mr. Mohamad Seif



OBJECTIVES

Study the decay of the current in RL circuit

ACADEMY

Decay of the current:
$$[u_g = 0] \dots \frac{1}{2} < t < T$$

Apply law of addition of voltages:

$$u_{AC} = u_{AB} + u_{BC}$$

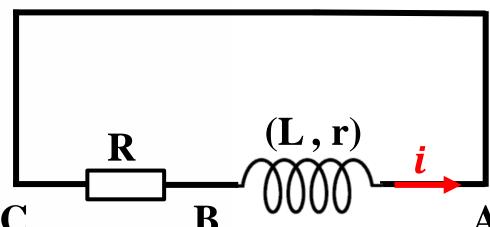
$$0 = u_{AB} + u_{BC}$$

$$0 = ri + L\frac{di}{dt} + Ri$$

$$0 = (R+r)i + L\frac{di}{dt}$$

$$R_{eq} = (R+r)$$

Differential equation in terms of current i



The solution of the differential equation is:

$$i = Ie^{-\frac{\iota}{\tau}}$$

Where
$$\tau = \frac{L}{(r+R)}$$
 And $I = \frac{E}{(r+R)}$

And
$$I = \frac{E}{(r+R)}$$

At
$$t = 0$$
:

$$i = I \cdot e^{-\frac{0}{\tau}}$$

$$Ai = A(e^0) = MY$$

$$i = I$$

At
$$t = \tau$$

$$i = I.e^{-\frac{\tau}{\tau}}$$

$$i = I.e^{-1}$$



$$i = 0.37I$$

$$i = 0.37I = 37\%I$$

 $t = \tau$: Time constant is the interval of time after which the current reaches 37% of its max value during decay process.

At
$$t = 5\tau$$

$$u_C = I.e^{-5}$$

$$i = I.e^{-\frac{5\tau}{\tau}}$$

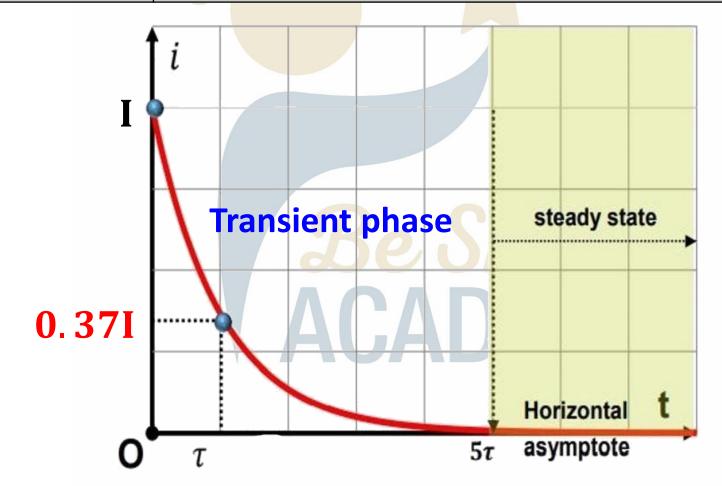


$$i = 0.001I$$

 $i \approx 0$

At $t = 5\tau$, the steady state is attained and the coil acts as a resistor of resistance r.

t = 0	t = au	t=5 au
i = I	i = 0.37I	i = 0



Time constant using tangent method:

First method:

Draw tangent to i = f(t) at t = 0

The slope of the tangent at t = 0 is $\frac{di}{dt}$

$$\dot{t} = I.e^{-\frac{t}{\tau}}$$

$$\frac{di}{dt} = I$$

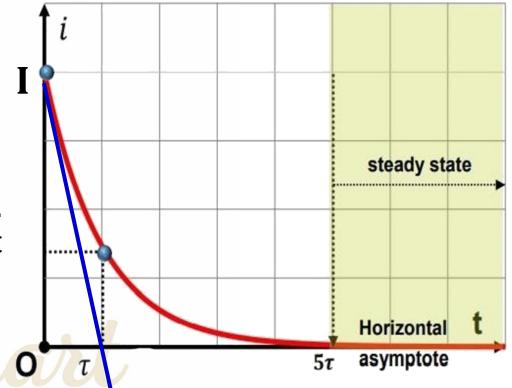
$$i = I.e^{-\frac{t}{\tau}}$$
 $\Rightarrow \frac{di}{dt} = I\left[\frac{-1}{\tau}\right]e^{-\frac{t}{\tau}}$

At
$$t = 0$$
:

$$\frac{di}{dt} = I \left| \frac{-1}{\tau} \right|$$



$$\frac{di}{dt} = I \left[\frac{-1}{\tau} \right] e^{-\frac{0}{\tau}} \implies \text{slope} = \frac{di}{dt} = \frac{-I}{\tau}$$



In
$$\triangle$$
 OHK: $tan\alpha = \frac{OK}{OH} = \frac{I}{OH}$ If K $slope = -tan\alpha$ $\frac{-I}{\tau} = -\frac{I}{OH}$ $\tau = OH$

Therefore, the tangent to i at t=0 meets the horizontal asymptote of equation $i_1=0$ at a point of abscissa is τ

Second method:

The equation of tangent to i at t=0 I

is: i = at

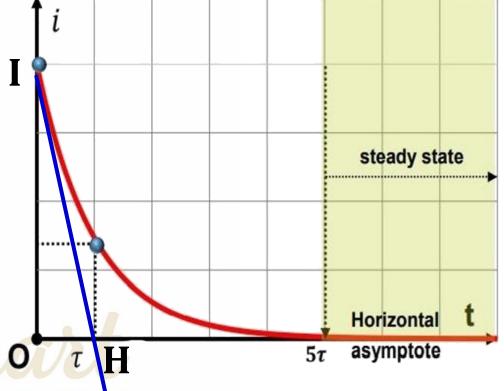
Where a is the slope of the tangent

at t = 0

$$i = I.e^{-\frac{t}{\tau}}$$

$$\frac{di}{dt} = I \left| \frac{-1}{\tau} \right| \epsilon$$

$$i = I.e^{-\frac{t}{\tau}}$$
 $\Rightarrow \frac{di}{dt} = I\left[\frac{-1}{\tau}\right]e^{-\frac{t}{\tau}}$



At
$$t = 0$$
:

$$\frac{di}{dt} = I \begin{bmatrix} -1 \\ \tau \end{bmatrix} e^{-\frac{0}{\tau}}$$

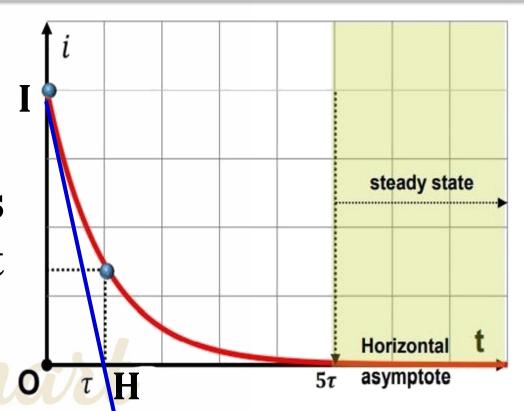


$$slope = \frac{a\iota}{dt} = \frac{-\iota}{\tau}$$

The equation of tangent is: i = at

$$i = -\frac{I}{\tau}.t$$

The tangent meets to i = f(t) meets the horizontal asymptote $i_1 = 0$ at point of of abscissa τ



Determination of I and τ in terms of r, R, and L

$$R_{eq}i + L\frac{di}{dt} = 0$$

$$i = Ie^{-\frac{t}{\tau}}$$

$$rac{di}{dt} = -rac{I}{ au}$$
 . $e^{-rac{t}{ au}}$

 $\frac{di}{dt} = -\frac{I}{\tau} \cdot e^{-\frac{t}{\tau}}$ Substitute i and $\frac{di}{dt}$ in differential equation.

$$R_{eq}I.e^{\frac{t}{\tau}}-L.\frac{t}{\tau}.e^{\frac{t}{\tau}}=0$$

$$R_{eq}Ie^{-\frac{t}{\tau}} = \frac{LI}{\tau}.e^{-\frac{t}{\tau}}$$

$$R_{eq}Ie^{-\frac{t}{\tau}} = \frac{LI}{\tau}.e^{-\frac{t}{\tau}}$$

By identification we get:

$$R_{eq}I = \frac{LI}{\tau}$$

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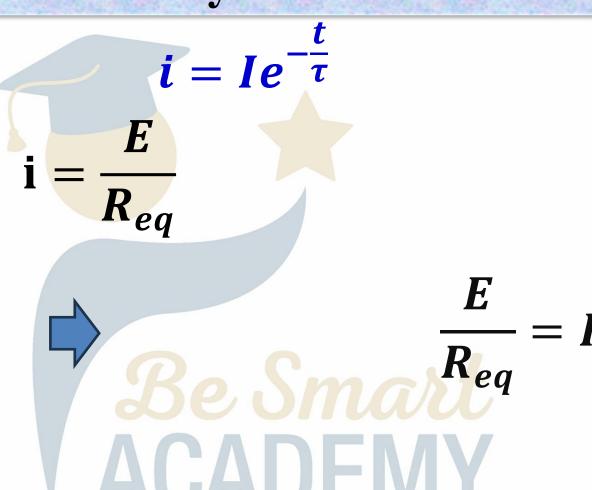
$$R_{eq}I = \frac{LI}{\tau}$$

$$ACAL TEMY$$

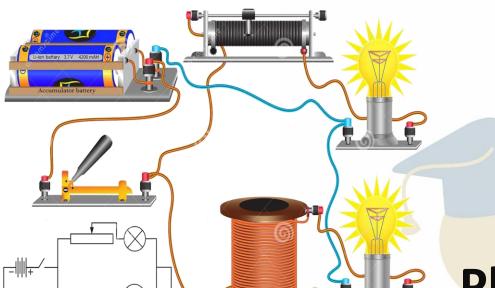
$$\tau = \frac{R_{eq}I}{R_{eq}I}$$

At
$$t = 0$$
:

$$\frac{E}{R_{eq}} = Ie^0$$

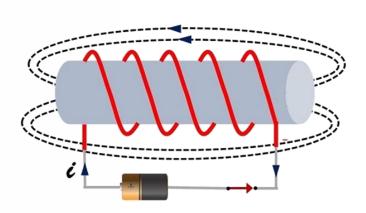






What is Self Induction?





Unit Two – Electricity

Chapter 9 – Self Induction

ACADEMY

Prepared & Presented by: Mr. Mohamad Seif



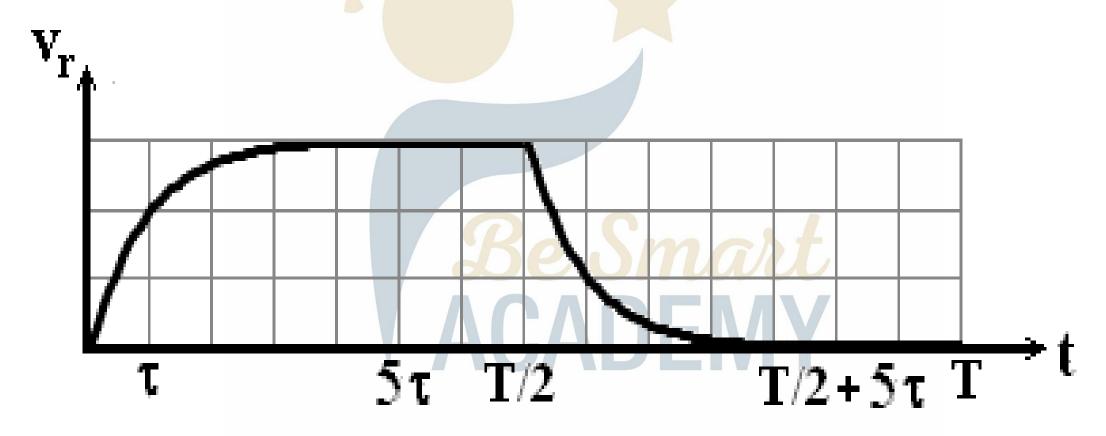
OBJECTIVES

1 Study the graphs of the two intervals

Study the sparks due to switching off a circuit

Graph of the two intervals

In the case if T/2 (of the L.F.G) > 5 τ , the graph is similar to that obtained by the oscilloscope



Graph of the two intervals

If T/2 (of the L.F.G) $< 5\tau$, the voltage will not have sufficient time to reach the value V_r , because at t = T/2 the e.m.f of the L.F.G becomes zero and V_r begins to decrease.

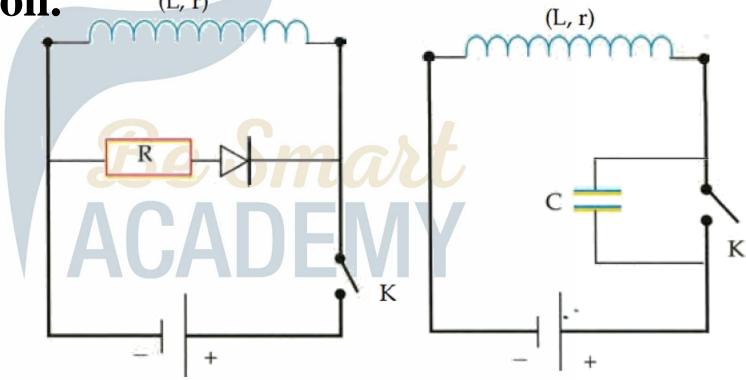
Also, V_r will not have sufficient time to reach the value 0, because at t = T the e.m.f of the L.F.G increases suddenly to

E and V_r begins to increase. V_r

Sparks due to switching off a circuit

When a circuit of large inductance is opened abruptly, a spark appears at the switch contacts due to a very high voltage across its terminals originating from the high e.m.f.

"e" induced in the coil.



Sparks due to switching off a circuit

Similarly, strong sparks are produced when we suddenly disconnect domestic appliances, including electric motors and transformers (those including coils) from the mains.

This phenomenon is interpreted by the presence of an excess voltage across the terminals of the switch at the instant when the circuit is opening.

The sparks produced may in the long turn destroy the contacts of the switches and can even destroy the brushes in generators and motors or break the insulator of the coil.

To avoid such damage, we must protect the switch.

